# LECTURE 101

## PULLEYS, CABLES AND FRICTION

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# Pulleys, Cables and Friction

### Nomenclature

| | | | |
|---|---|---|---|
| $F$ | force | lbf | N |
| $D$ | diameter | ft | m |
| $g$ | acceleration due to gravity | ft/sec$^2$ | m/sec$^2$ |
| $g_c$ | gravitational constant | lbm-ft/lbf-sec$^2$ | - |
| $m$ | mass | lbm | kg |
| n | number of sheaves | - | - |
| N | normal force | lbf | N |
| P | power | ft-lbf/sec | W |
| $r$ | radius | ft | m |
| R | reaction force | lbf | N |
| T | tension | lbf | N |
| T | torque | ft-lbf | N-m |
| $v$ | velocity | ft/sec | m/s |
| $W$ | weight | lbf | N |

### SYMBOLS

| | | | |
|---|---|---|---|
| $\theta$ | wrap angle | rad | rad |
| $\mu$ | coefficient of friction | - | - |
| Ǿ | inclination angle | deg | deg |

### SUBSCRIPTS

| | |
|---|---|
| s | static |
| f | friction |
| t | tangential |

## Pulleys

A pulley (also known as a sheave) is used to change the direction of an applied tensile force. A series of pulleys working together (known as block and tackle) can also provide pulley advantage (i.e., mechanical advantage). If the pulley is attached by a bracket to a fixed location, it is said to be a fixed pulley. If the pulley attached to a load, or if the pulley is free to move, it is known as a free pulley.

Most simple problems disregard friction and assume that all rope (fiber ropes, wire ropes, chains, belts, etc.) are parallel. In such cases, the pulley advantage is equal to the number of ropes coming to and going from the load-carrying pulley. The diameters of the pulleys are not factors in calculating the pulley advantage.

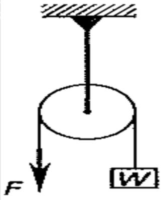
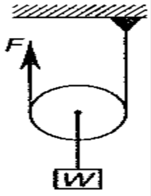
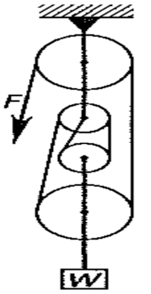
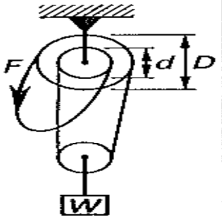
| | fixed sheave | free sheave | ordinary pulley block ($n$ sheaves) | differential pulley block |
|---|---|---|---|---|
| |  |  |  |  |
| $F_{\text{ideal}}$ | $W$ | $\frac{W}{2}$ | $\frac{W}{n}$ | $\frac{W}{2}\left(1-\frac{d}{D}\right)$ |

**Figure 1 Mechanical Advantage of Rope-Operated Machines**

## Cables

An ideal cable is assumed to be completely flexible, massless, and incapable of elongation; therefore, it acts as an axial tension member between points of concentrated loading. In fact, the term tension or tensile force is commonly used in place of member force when dealing cables.

The methods of joints and sections used in truss analysis can be used to determine the tensions in cables carrying concentrated loads. After separating the reactions into x- and y-components, it is particularly useful to sum moments about one of the reaction points. All cables will be found to be in tension, and (with vertical loads only) the horizontal tension component will be the same in all cable segments. Unlike the case of a rope passing over a series of pulleys, however, the total tension in the cable will not be the same in every cable segment.

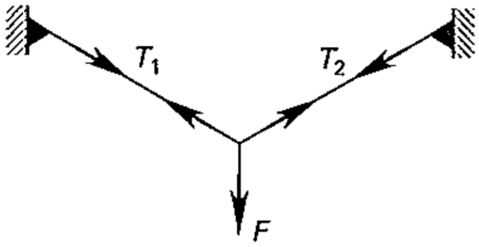


**Figure 2 Cable with Concentrated Load**

## Friction

Friction is a force that always resists motion or impending motion. It always acts parallel to the contacting surfaces. The frictional force, F, exerted on a stationary body is known as static friction, Coulomb friction, and fluid friction. If the body is moving, the friction is known as dynamic friction and is less than the static friction.

The actual magnitude of the frictional force depends on the normal force, N, and the coefficient of friction, μ, between the body and the surface.

$$F=\mu N \qquad 1$$

For a body resting on a horizontal surface, the normal force is the weight of the body.

$$N = mg \qquad \text{[SI]} \qquad 2a$$

$$N = \frac{mg}{g_c} \qquad \text{[U.S.]} \qquad 2b$$

If a body rests on a plane inclined at an angle Ǿ from the horizontal, the normal force is

$$N = mg \cos \emptyset \qquad \text{[SI]} \qquad 3a$$

$$N = \frac{mg \cos \emptyset}{g_c} \qquad \text{[U.S.]} \qquad 3b$$

## Belt friction

Friction between a belt, rope, or band wrapped around a pulley or sheave is responsible for the transfer of torque. Except when stationary, one side of the belt (the tight side) will have a higher tension than the other (the slack side). The basic relationship between the belt tensions and the coefficient of friction neglects centrifugal effects and is given by Eq. 4. $F_1$ is the tension on the tight side (direction of movement); $F_2$ is the tension on the other side. The angle of wrap, ϴ, must be expressed in radians.

$$F_1 = F_2 e^{\mu\theta} \qquad 4$$

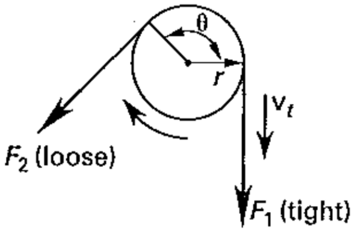


**Figure 3 Belt Friction**

The net transmitted torque is:

$$T = (F_1 - F_2)r \qquad 5$$

The power transmitted to a belt running at tangential velocity $v_t$ is

$$P = (F_1 - F_2)v_t \qquad 6$$

## Square Screw Threads

A power screw changes angular position into linear position (i.e., changes in rotary motion into traversing motion). The linear positioning can be horizontal (as in vices and lathes) or vertical (as in a jacks). Square, Acme, and 10-degree modified screw threads are commonly used in a power screws. A square screw thread is shown in Fig. 4.

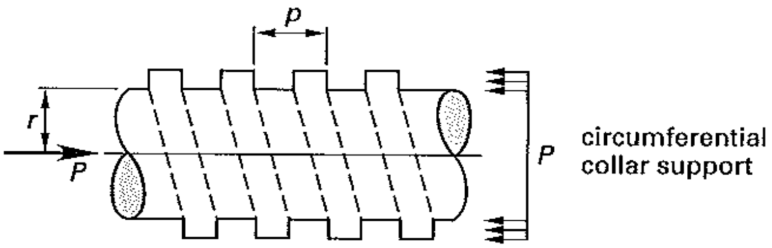


**Figure 4 Square Screw Thread**

A square screw thread is designated by a mean radius, r, pitch, p, and a pitch angle, $\alpha$. The pitch, p, is the distance between corresponding points on a thread. The lead is the distance the screw advances each revolution. Often, double and triple-threaded screws are used. The lead is one, two, or three times the pitch for single-, double-, and triple-threaded screws, respectively.

$$p = 2\pi r \ \tan\alpha \tag{7}$$

The coefficient of friction, $\mu$, between the threads can be designated directly or by way of a thread friction angle, $\emptyset$.

$$\mu = \tan\emptyset \tag{8}$$

The torque or external moment, M, required to turn a square screw in motion against an axial force, P (i.e., "raise" the load), is

$$M = Pr\tan(\alpha + \emptyset) \tag{9}$$

The torque required to turn the screw in motion in the direction of the applied axial force (i.e., "lower" the load) is given by Eq. 10. If the torque is zero or negative (as it would be if the lead is large or friction is low), then the screw is not self-locking and the load will lower by itself, causing the screw to spin (i.e., it will "overhaul"). The screw will be self-locking when tan $\alpha \leq \mu$.

$$M = Pr\tan(\alpha - \emptyset) \tag{10}$$

The torque calculated in Eqs. 9 and 10 is required to overcome thread friction and to raise the lead (i.e., axially compress the screw). Typically, only 10-15 percent of the torque goes into axial compression of the screw. The remainder is used to overcome friction. The mechanical efficiency of the screw is the ratio of torque without friction to the torque with friction. The torque without friction can be calculated from Eq. 9 or 10 (depending on the travel direction) using Ǿ = 0.

$$\mu_m = \frac{T_f = 0}{T} \tag{11}$$

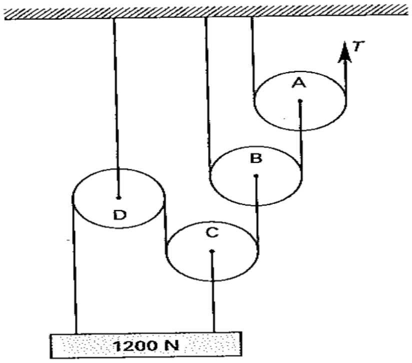
In the absence of an antifriction ring, an additional torque will be required to overcome friction in the collar. Since the collar is generally flat, the normal force is the jack load, P, for the purpose of calculating the frictional force.

$$T_{collar} = P\mu_{collar} r_{collar} \qquad 12$$

**Sample problems:**

Problem 1

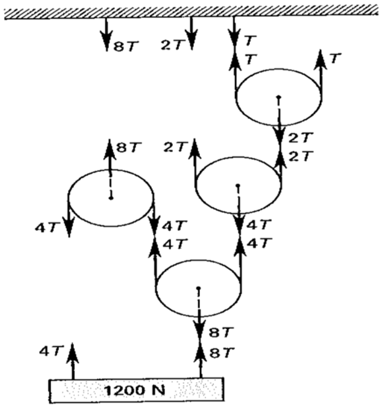
Find the tension, T, that must be applied to pulley A to lift the 1200 N weight.



a. 100 N
b. 300 N
c. 400 N
d. 600 N

*Solution:*

The free bodies of the system are shown.



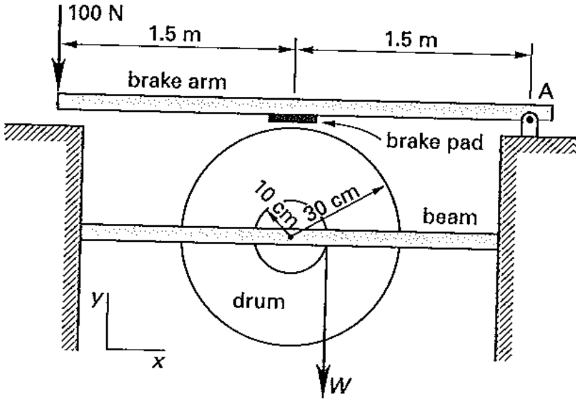
$$\sum F_y = 0$$

$$= -1200N + 4T + 8T$$

$$12T = 1200N$$

$T = 100N$ **(a)**

Problems 2 and 3 refer to the following illustration.



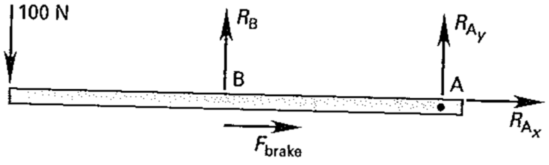
Problem 2

The coefficient of friction between the brake pad and drum is 0.3. assuming that the beam supporting the cable drum is rigid and more than adequate for the loads involved, what load, W, can be held stationary?

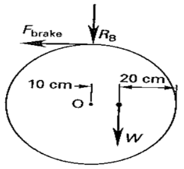
a. 33 N b. 90 N c. 100 N d. 180 N

*Solution:*

The free bodies of the brake arm and drum are



(a) free body of the brake arm



(b) free body of the drum

The brake reaction (normal force), $R_B$, is found by summing moments about the lever pivot point.

$$\begin{aligned} \sum M_A &= 0 \\ &= -(100N)(3m) + R_B(1.5m) \\ R_B &= 200N \\ F_{brake} &= \mu R_B = (0.3)((200N) \\ &= 60\ N \end{aligned}$$

From the free body of the drum,

$$\begin{aligned} \sum M_O &= 0 \\ &= W(0.1\ m) - (60N)(0.3m) \end{aligned}$$

$W = 180\ N$ [maximum without slipping] (d)

Problem 3

If W = 80 N, what are the reactions at point A?

a. 0i + 100 j N

b. 0i + 180 j N

c. 27i N + 100j N

d. -27i N - 100j N

*Solution*

Refer to the free-body diagram of the brake arm in Prob. 2.

$$F_{brake}(0.3m) = W(0.1m)$$

$$F_{brake} = \frac{W}{3} = \frac{80N}{3} = 26.67 \quad \text{[to the right]}$$

$$\sum F_x = 0$$

$$= F_{brake} + R_{Ax}$$

$$R_{Ax} = -F_{brake}$$

$$= -26.67\ N \qquad \text{[to the left]}$$

$$\sum F_y = 0$$

$$= -100N + +R_B + R_{Ay}$$

$$R_{Ay} = 100N - 200N$$

$$R = -27i\ N - 100j\ N$$